Global dynamic routing for scale-free networks

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Traffic is essential for many dynamic processes on networks. The efficient routing strategy [G. Yan, T. Zhou, B. Hu, Z. Q. Fu, and B. H. Wang, Phys. Rev. E **73**, 046108 (2006)] can reach a very high capacity of more than ten times of that with shortest path strategy. In this paper, we propose a global dynamic routing strategy for network systems based on the information of the queue length of nodes. Under this routing strategy, the traffic capacity is further improved. With time delay of updating node queue lengths and the corresponding paths, the system capacity remains constant, while the travel time for packets increases.

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I. INTRODUCTION

Dynamical properties of network traffic have attracted much attention from physical and engineering communities. The prototypes include the transfer of packets in the Internet, the flying of airplanes between airports, the motion of vehicles in urban network, the migration of carbon in biosystems, and so on. Since the discovery of small-world phenomenon [1] and scale-free property [2], it is widely proved that the topology and degree distribution of networks have profound effects on the processes taking place on these networks, including traffic flow [3–6].

The traffic jamming transition in the Internet was observed and analyzed in 1996 [7]. In the past few years, the phase-transition phenomena [8–11], the scaling of traffic fluctuations [12–15], and the routing strategies [16–33] of traffic dynamics have been widely studied. Compared with the high cost of changing the infrastructure, developing a better routing strategy is usually preferable to enhance the network capacity. Nowadays, the shortest path strategy is widely used in different systems. However, it often leads to the failure of hub routers with high degree and betweenness. In this light, some new routing strategies have been suggested, such as the efficient routing strategy [21], the integration of shortest path and local information [22], the next-nearest-neighbor searching strategy [29], the local routing strategy [31,32], and so on.

In the efficient routing strategy [21], the path between any nodes i (source) and j (destination) is defined as the path in which the sum degree of nodes is a minimum. It is denote as

$$\mathcal{P}_{ij} = \min \sum_{m=0}^{l} k(x_m)^{\beta}, \tag{1}$$

where $k(x_m)$ is the degree of node x_m, l is the path length, and β is a tunable parameter. When β is set to 1, the largest network capacity can be achieved, where packets are propelled to the peripheral of the network. The efficient path can achieve a very high capacity for the network, which can be ten times of that with the shortest path strategy. In the local routing strategy [31], the packets are forwarded by the local information of neighbors' degree,

$$\Pi_{l\to i} = \frac{k_i^{\alpha}}{\sum_j k_j^{\alpha}}. \tag{2}$$

It is also proved that the best routing strategy emerges at $\alpha = -1$, where the packets are forced to select the low-degree nodes. In the routing strategy based on the integration of static and dynamic information [32], the queue length of neighboring nodes is taken into consideration,

$$\Pi_{l\to i} = \frac{k_i(n_i+1)^{\beta}}{\sum_j k_j(n_j+1)^{\beta}}, \tag{3}$$

where n_i is the queue length of the neighboring node i. It is shown that the hub nodes play an important role in the traffic process.

In this paper, we propose that an efficient network routing strategy should not only consider the topology of the network, but also the effects of the queue length of nodes. We introduce a global dynamic routing strategy for the networks. In this routing strategy, the packets are delivered along the path in which the sum queue length of nodes is a minimum. Numerical results show that the system capacity can be further improved to almost two times of that with the efficient routing strategy [21]. The effects of the strategy on the efficiency of the scale-free traffic system are discussed in detail. The effects of the path update delay are also investigated.

The paper is organized as follows. In the following section, we describe the network model and traffic rules in detail. In Sec. III, the simulation results are presented and discussed. In the last section, the work is concluded.

II. TRAFFIC MODEL

Recent studies indicate that many communication systems such as the Internet and the World Wide Web are not homogeneous as random or regular networks, but heterogeneous with a degree distribution following the power-law distribution $P(k) = k^{-\gamma}$. The Barabási-Albert (BA) model [2] is a well-known model which can generate networks with a power-law degree distribution. Without lose of generality, we

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construct the underlying network structure by the BA network model. In this model, starting from m_0 fully connected nodes, a new node with m edges ($m \le m_0$) is added to the existing graph at each time step according to preferential attachment, i.e., the probability Π_i of being connected to the existing node i is proportional to the degree k_i of the node,

$$\Pi_i = \frac{k_i}{\sum_j k_j}. \quad (4)$$

The traffic model is described as follows: at each time step, R packets enter the system with randomly chosen sources and destinations. One node can deliver at most C (here, we set $C=1.0$ for each node) packets to its neighboring nodes. From the many paths between the source and destination, we selected the path in which the sum of node queue lengths is a minimum. Therefore, the path between nodes i (source) and j (destination) can be denoted as

$$\mathcal{P}_{ij} = \min \sum_{m=0}^{l} [1 + n(x_m)], \quad (5)$$

where $n(x_m)$ is the queue length of node x_m and l is the path length. The maximal queue length of each node is assumed to be unlimited and the first-in-fist-out discipline is applied at each queue. Once a packet arrives at its destination, it is removed from the system.

In order to describe the phase transition of traffic flow in the network, we use the order parameter [8]

$$\eta(R) = \lim_{t\to\infty} \frac{C}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}, \quad (6)$$

where $\Delta N_p = N_p(t+\Delta t) - N_p(t)$, $\langle \cdots \rangle$ indicates the average over time windows of width Δt, and $N_p(t)$ is the total number of packets within the network at time t. The order parameter represents the balance between the inflow and outflow of packets. In the free flow state, due to the balance of created and removed packets, η is around zero. With increasing packet generation rate R, there will be a critical value of R_c that characterizes the phase transition from free flow to congestion. When R exceeds R_c, the packets accumulate continuously in the network, and η will become a constant larger than zero. The network capacity can be measured by the maximal generating rate R_c at the phase-transition point.

III. SIMULATION RESULTS

Figures 1(a)–1(c) show the evolution of the total packet number N_p under shortest, efficient, and global dynamic paths with the network size $N=500$ and average degree $\langle k \rangle = 4$. Figure 1(d) compares the relation of order parameter η vs R under the three routing strategies. One can see that the network capacity under the shortest path routing strategy is $R_c=3$. The efficient routing strategy has $R_c=20$. The global dynamic routing strategy reaches a very high capacity of $R_c=41$, which is more than double of the efficient routing strategy. As far as we know, the global dynamic routing strategy can achieve the highest traffic capacity.

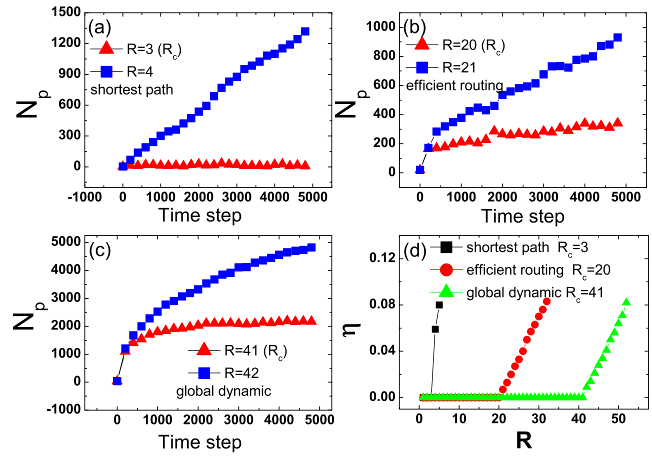

FIG. 1. (Color online) Evolution of packet number in the network under different routing strategies. (a) Shortest path routing strategy, (b) efficient routing strategy, and (c) global dynamic routing strategy. (d) The order parameter η vs R under the three routing strategies.

In Fig. 2, we show the relation of network's capacity R_c vs the average degree $\langle k \rangle$, and vs the network size N under three routing strategies. Figure 2(a) shows that with the same network size, the network's capacity increases with the average degree $\langle k \rangle$ under three routing strategies. The rank of network capacities is global dynamic routing $>$ efficient routing $>$ shortest path routing. In Fig. 2(b), one can also see that the network's capacity increases with the increasing of the network size N. Again, with the same average degree or network size, the network's capacity under the global dynamic routing strategy is the largest.

Because the node queue length changes from time to time, it is computation consuming to find the global dynamic paths in each step. Therefore, it is important to introduce a time delay (δT) for the update of the global queue information and the corresponding paths.

Figure 3 shows the evolution of the total packet number N_p for different R with a time delay of $\delta T=20$. Although the dynamic paths are updated every 20 time steps, the capacity of network still remains at $R_c=41$. Figure 4 shows the evolution of the total packet number N_p for other values of δT. One can see that the capacity of network under different δT

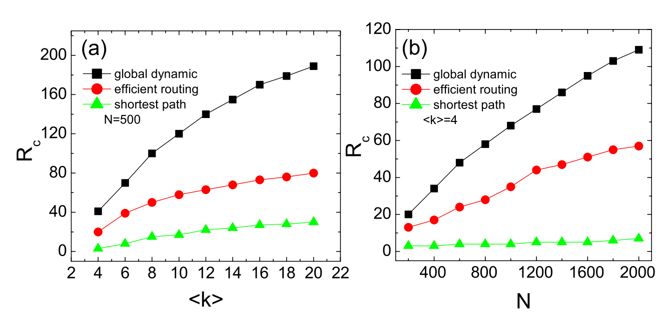

FIG. 2. (Color online) (a) The network capacity R_c vs average degree $\langle k \rangle$ with the same network size $N=500$ under the three different routing strategies. (b) The network capacity R_c vs network size N with the same average degree $\langle k \rangle = 4$ under the three routing strategies.

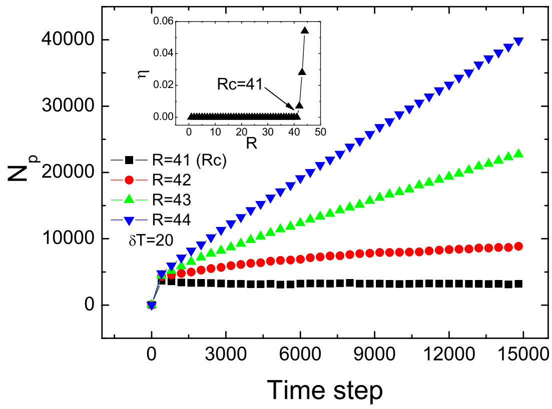

FIG. 3. (Color online) Evolution of packet number N_p for different R with time delay of $\delta T=20$. The inset depicts the order parameter η vs R with $\delta T=20$. The network capacity remains at $R_c=41$. Other parameters are network size $N=500$ and average degree $\langle k\rangle=4$.

remains the same ($R_c=41$). This demonstrates that the capacity of network is independent of δT. However, one can see that the saturate value of N_p increased with the increase in δT, as shown in the inset of Fig. 4. This indicates that the queue length of nodes in the system will increase, although the network capacity remains constant at $R_c=41$.

In Fig. 4, one can see that N_p first increase to a maximum and then decrease to a balanced state. This phenomenon is more obvious with larger values of δT. This can be explained as follows. At the beginning of transportation, there is no packets in each node $[n(x_m)=0]$. Thus, the packets are delivered following the shortest path routing strategy until the time reaches $T=\delta T$. Because the network capacity of the shortest path routing is only $R_c=3$, N_p will increase rapidly as in the congested state. When $T=\delta T$, the global dynamic routing strategy begins to take action, so that N_p decreases to a balanced state.

Figure 5 shows the dependence of the average queue length $n(k)$ on the degree k with different δT in the free flow state. For different δT, $n(k)$ follows a power law of $n(k)$

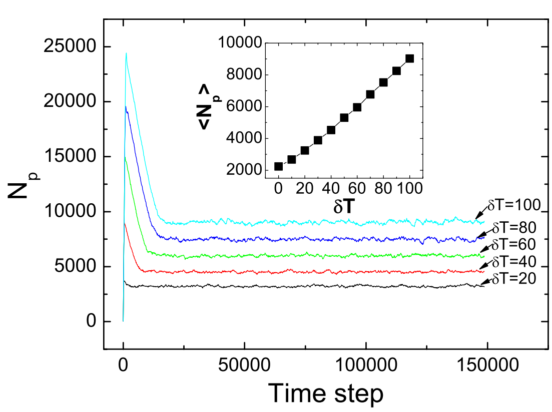

FIG. 4. (Color online) Evolution of N_p for different time delay with $R=R_c=41$. For all δT, the network capacity remains at $R_c=41$. The inset depicts the saturate value of N_p at the balanced period vs δT under $R=R_c=41$. Other parameters are network size $N=500$ and average degree $\langle k\rangle=4$.

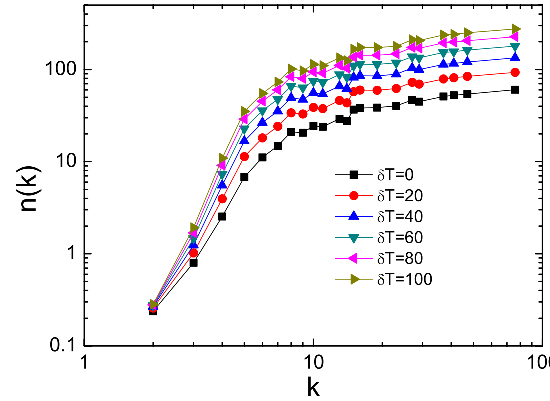

FIG. 5. (Color online) Average packet number in nodes $n(k)$ vs node degree k with different δT. Network size $N=500$ and average degree $\langle k\rangle=4$.

$\sim k^{\gamma}$ with exponent $\gamma>0$. This indicates that the hub nodes are more burdened with this routing strategy. Moreover, for each k, the value of $n(k)$ increases with δT. This is in agreement with that the total packet number increased with δT under the same R (Fig. 4, inset).

Then we investigated the probability distribution of the packet travel time for different δT in the free flow state. Packet's travel time is an important factor for characterizing the network's behavior. The travel time is the time that the packet spends traveling from the source to destination. In Fig. 6, one can see that the probability of traveling time approximately follows a Poisson distribution under different δT. The travel time corresponding to the peak of distribution will increase with δT. This indicates that the packets need to travel for longer times if the global dynamic paths are updated with the time delay.

Figure 7 shows the average travel time and the waiting time for different δT. The waiting time is the time that the packets spend in a queue waiting for the delivery. One can see that both the average travel time and the average waiting time increase almost linearly with δT. This phenomenon is consistent with the result of Fig. 6. With the same R, when δT increases, the queue length in each node increases (see Fig. 5), so the waiting time of packets also increases. As a

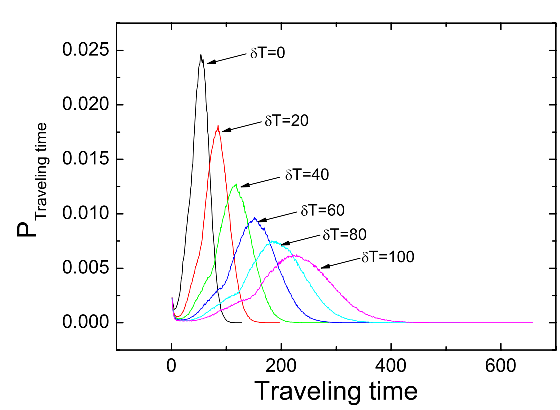

FIG. 6. (Color online) The probability distribution of travel time for different δT. Network parameters are $N=500$ and average degree $\langle k\rangle=4$.

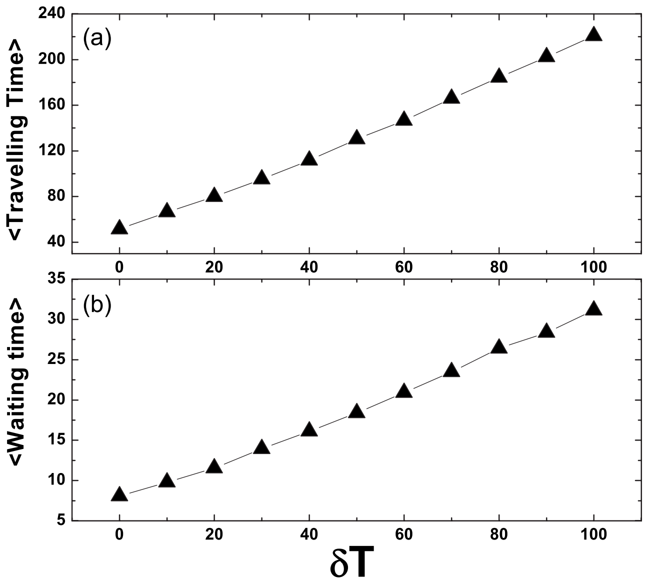

FIG. 7. (a) Average travel time vs δT with network size $N=500$ and average degree $\langle k\rangle=4$. (b) Average waiting time vs δT with network size $N=500$ and average degree $\langle k\rangle=4$.

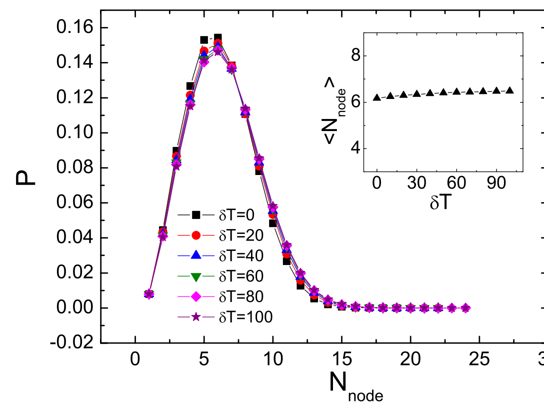

FIG. 8. (Color online) The probability distribution of path length for different δT with network size $N=500$ and average degree $\langle k\rangle=4$. The inset depicts the average value of path length vs δT.

result, the average traveling time will also increase. This indicates that with the introduction of the time delay, the system's capacity remains the same at the cost of increasing the packets' travel time.

Finally, we investigate the probability distribution of the path length for different δT in the free flow state. The path length is defined as the number of nodes that the packet travels from the source to destination. As shown in Fig. 8, the probability of the path length approximately follows a Poisson distribution for different δT. From the inset of Fig. 8, one can also see that the average path lengths increase very slightly with δT. Therefore, the increment of packets' travel time comes mainly from the increment of the waiting time in each queue along the path, but not from the increment of the path length.

IV. CONCLUSION

In summary, we propose a global dynamic routing strategy for scale-free networks. Under this strategy, the traffic capacity is much larger than other previously known routing strategies. Compared with the efficient routing strategy [21] and the local routing strategy [31], the global dynamic routing strategy differs from their mechanism of propelling the packet to use the peripheral nodes of the network. The global dynamic routing strategy still encourages us to use the hub nodes (see Fig. 5), but it can achieve a much higher capacity for the system. We also investigate the delay effects of the path update. With the increment of the update delay, the network's traffic capacity remains the same, but the total packet number in system, the traveling time, and the waiting time will increase. The delay of the path update will not affect the overall capacity and the average path length. The increment of the travel time comes mainly from the increase in the waiting time in the queues.

Since the traffic process is crucial for many industrial and communication systems, the global dynamic strategy can be useful for the design and optimization of routing strategies for these systems, including the Internet, the World Wide Web, the urban transportation system [34–36], the power grid, the airway system, and so on. The study can also be interesting from a theoretical point of view, as an upper bound for the efficiency.

In the future work, we will apply the global dynamic routing strategy in other network structures. It is expected the traffic capacity of these networks will be enhanced with this routing strategy.

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